# What do quantum particles do, being under potential barrier? Tunnelling time. A Virtual Experiment Standpoint.

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**Abstract**

Addressed, mainly: postgraduates and related readers.

Subject: Given two classical mechanical $1D$-moving particles (material points), with identical initial data, one of those particles given free and another given to pass through a symmetrical force-barrier, a retardation effect is observed: After the barrier has been passed over, the second particle moves with the same velocity as the free particle, but spacially is retarded with respect to the latter. If the "non-free" particle moves through a potential well, then the retarded particle is the free particle.

The question is. What phenomena of a similar kind could one find, if the $1D$-moving particles were quantum ones? And just what do quantum particles do, being under potential barrier? I here say "quantum" in a mathematical sense: "Schroedinger".

To answer the question, I had constructed some suitable Virtual Devices (Java applets) and then for some time experimented various situations. I detected some phenomena that I could name "a retardation", however I doubt whether that term is proper. Some of those Java applets are available at `http://choroszavin.narod.ru/vlab/index-2.htm`

Keywords: Scattering, Finite Rank Perturbations.

Given two classical mechanical $1D$-moving particles (material points), with identical initial data, one of those particles given free and another given to pass through a symmetrical force-barrier:

$$F = \begin{cases} -F_0 & , \quad x \in [-w/2 + x_0, x_0] \\ F_0 & , \quad x \in [x_0, x_0 + w/2] \end{cases} \qquad ,$$

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The question is. What phenomena of a similar kind could one find, if the $1D$-moving particles were quantum ones? And just what do quantum particles do, being under potential barrier? I here say "quantum" in a mathematical sense: "Schroedinger".

To answer the question, I had constructed some suitable Virtual Devices (Java applets) and then for some time experimented various situations. I detected some phenomena that I could name "a retardation", however I doubt whether that term is proper. As for the most certain and definite manifestations of the retardation, I had untill now detected them by those of my Virtual Devices, which I had constructed on principles of the stationary scattering theory.

Mathematically they are based on an analysis of

$$i\partial_t \psi(t,x) \quad = \quad -\partial_x^2 \psi(t,x) + \sum_a \alpha_a \delta(x - x_a) \psi(t,x)(x_a) \, .$$

A solution (an eigensolution) is of the form

$$\psi(t,x) \quad = \quad e^{-i\omega t} \widetilde{w}_k(x) \, , \quad \omega = k^2 \, ,$$

$$\widetilde{w}_k(x) \quad = \quad e^{ikx} + \sum_a coef_a \cdot e^{i|k| \cdot |x - x_a|} \, ,$$

and any finite linear combination

$$\psi(t,x) \quad = \quad \sum_m A_m e^{i\phi_m} e^{-i\omega_m t} \widetilde{w}_{k_m}(x) \, ,$$

is a solution as well. Of course, $\{coef_a\}_a$ depend on $\{x_a\}_a$, $\{\alpha_a\}_a$, and $k$ (resp., $k_m$). The stationary scattering theory says that if the "free" system behaves as

$$\psi_{free}(t,x) \quad = \quad \sum_m A_m e^{i\phi_m} e^{-i\omega_m t} e^{ik_m x} \, ,$$

then the "non-free" will behave as

$$\psi_{non-free}(t,x) \quad = \quad \sum_m A_m e^{i\phi_m} e^{-i\omega_m t} \widetilde{w}_{k_m}(x) \, .$$

I took, mainly,

$$\psi_{free}(t,x) \quad = \quad \sum_m e^{-i\omega_m t} e^{ik_m x} \, ,$$

and observed the graphs of

$$|\psi_{free}(t,x)|^2 \, , \quad |\psi_{non-free}(t,x)|^2 \, ,$$

especially, ones of

$$|\psi_{free}(0,x)|^2 \, , \quad |\psi_{non-free}(0,x)|^2 \, .$$

In such a case

$$|\psi_{free}(0,x)|^2$$

looks like a series of quasiperiodic relatively localized excitations. The structure of

$$|\psi_{non-free}(0,x)|^2$$

looks much more complicated, although sometime it could be specified as a set of "excitations" and there I can not formulate my impressions more defintely.

As for the basic theme of my paper, I found that:

the less was the effective width of the "free"-excitations with respect to the region of

$$\sum_a \alpha_a \delta(x - x_a)\psi(t,x)(x_a) \, ,$$

—i.e., with respect to

$$\max\{x_a\}_a - \min\{x_a\}_a$$

,— and the less were the distances between scattering centres with respect to the effective width of the "free"-exitations, the more explicitly I observed a phenomenon which I could name "retardation".

Some of my Virtual Laboratories (Java applets) where I observed described phenomena certainly and explicitly are located at `http://choroszavin.narod.ru/vlab/index-2.htm`

An instruction: to observe a retardation phenomemon move the scrollbar named *cc_b*

As for "I doubt whether that term is proper", another instruction is: set `n_de_las_ondas=1` and repeat measurements.

**Notes:**

The basic theme of the paper is generated by a theme of the papers of N.L. Chuprikov. The philosophy of the computations has associated with the paper [10] of S.R. Foguel and my papers [6]-[9].

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# References

[1] N. L. Chuprikov. Wave functions and characteristic times for transmission and reflection
arXiv:quant-ph/0405028 v3 13 Nov 2004

[2] N. L. Chuprikov. Dwell times for transmission and reflection
arXiv:quant-ph/0502073 v3 14 Jul 2005

[3] N. L. Chuprikov. Temporal aspects of tunneling: an alternative view
arXiv:quant-ph/0511024 v1 3 Nov 2005

[4] N. L. Chuprikov. One-dimensional completed scattering: wave functions and tunneling times for transmission and reflection
arXiv:quant-ph/0602172 v1 21 Feb 2006

[5] N. L. Chuprikov. Quantum Mechanics without Nonlocality
arXiv:quant-ph/0602173 v1 21 Feb 2006

[6] S.A. Choroszavin. An Interaction of An Oscillator with An One-Dimensional Scalar Field. Simple Exactly Solvable Models based on Finite Rank Perturbations Methods. I: D'Alembert-Kirchhoff-like formulae
arXiv:math.DS/0301167 v6 18 Nov 2003

[7] S.A. Choroszavin. An Interaction of An Oscillator with An One-Dimensional Scalar Field. Simple Exactly Solvable Models based on Finite Rank Perturbations Methods. II: Resolvents formulae
arXiv:math-ph/0302038 v3 16 Mar 2003

[8] S.A. Choroszavin. The Ground, a String, Two Elastic Springs. Simple Exactly Solvable Models of One-Dimensional Scalar Fields with Concentrated Factors.
arXiv:math-ph/0311045 v5 20 Oct 2004

[9] S.A. Choroszavin. Notes on Rank-One Perturbed Resolvent.
arXiv:math-ph/0312016 v3 20 Dec 2003

[10] S.R. Foguel, *Finite Dimensional Perturbations in Banach Spaces*, American Journal of Mathematics, Volume 82, Issue 2 ( Apr., 1960 ), 260-270

[11] W. Jung. Multiple Reflections in One-Dimensional Quantum Scattering (60K, LaTeX) Dec 31, 97
mp_arc 97-659,
`http://www.ma.utexas.edu/mp_arc/`
for the latest version, visit `http://www.iram.rwth-aachen.de/~jung`

[RS] M. Reed , B. Simon, *Scattering Theory*, Methods of Modern Mathematical Physics, vol 3, - N.Y.: Academic Press, 1979.